DIFFRACTIVE PHOTON DISSOCIATION
IN THE SATURATION REGIME

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Using the Good and Walker picture, we derive a simple formula for diffractive dissociation that can apply to recent data collected at HERA in the low Q^2 regime.

Deep inelastic scattering of a photon of virtuality Q off a given target can be viewed as the photon splitting into a quark-antiquark pair of size $r \sim 1/Q$, and subsequently scattering off the target through a further quantum fluctuation. This interpretation is valid in the leading logarithmic approximation of QCD when the total rapidity Y is very large, and in a frame in which Y is mostly carried by the projectile photon. The rapidity of the target in that frame is $Y_0 \ll Y$.

The initial $q\bar{q}$ pair is a color dipole. The higher Fock states built up from QCD radiation can also be interpreted as collections of dipoles [3]. That holds in the large N_c limit in which a gluon is essentially a zero-size $q\bar{q}$ state, as far as color is concerned.

There are typically two classes of events observed in experiments: either the target breaks up and the whole rapidity range is filled with decay products, or it remains essentially intact and an angular region empty of final state particles is seen in the detector. The latter events are called diffractive, and may represent up to 20% of all events. Among them, the ones that exhibit a final state made of the $q\bar{q}$ pair together with additional gluonic radiation in the photon fragmentation region are called *dissociative*. They are characterized experimentally by a large invariant mass $M_X \gg Q$ of the decay products of the photon.

The main goal of the work on which we report here [1] is to provide a simple derivation of the dissociative cross section valid also in the saturation regime, when effects due to unitarity corrections become sizeable, and to compare it to recent HERA data. We also argue that diffractive and total cross sections have the same rapidity dependence, and this comes about very naturally in our framework.

1 High energy behavior of deep-inelastic scattering observables

From a simple quantum mechanical calculation, one obtains that the total, elastic and dissociative cross sections for the scattering of a color dipole of size r at fixed

impact parameter b are given by

$$
\begin{aligned}
\frac{d\sigma_{tot}}{d^2b} &= 2\left(1 - \langle S(r,b)\rangle_{Y-Y_0}\right), \quad \frac{d\sigma_{el}}{d^2b} = \langle 1 - S(r,b)\rangle^2_{Y-Y_0} \\
\frac{d\sigma_{diss}}{d^2b} &= \langle S^2(r,b)\rangle_{Y-Y_0} - \langle S(r,Y)\rangle^2_{Y-Y_0} .
\end{aligned}
\tag{1}
$$

The diffractive cross section is the sum of the elastic and dissociative ones. $S(r,b)$ is the S-matrix element for the elastic scattering of a fixed partonic configuration of the dipole of size r at impact parameter b. It is a random variable, whose distribution is related to the probability distribution of a given Fock state of the initial $q\bar{q}$ pair.

The average $\langle.\rangle_{Y-Y_0}$ is taken over all possible partonic configurations of the initial dipole after evolution over a rapidity range $Y - Y_0$. The last formula above is due to Good and Walker [2], and is an identity between the dissociative cross section and the variance of the S-matrix for partonic states. The total cross section instead is related to the expectation value of the latter.

The evolution law for S with Y can be readily derived. Within the rapidity interval dy, a dipole of size r may split into two dipoles of size z, $r - z$ by emitting a gluon. That occurs with a probability proportional to dy, say $\lambda\,dy$. Let us denote by $\rho(z,r)$ the distribution of the sizes z, $r - z$ of the emitted dipoles. Then

$$
S(r) \xrightarrow[Y \to Y + dy]{} \begin{cases} S(r) & \text{with probability } 1 - \lambda\,dy \\ S(z)S(r-z) & \text{with probability } \lambda\,dy , \end{cases}
\tag{2}
$$

z being distributed according to $\rho(z,r)d^2z$. That evolution law implies a recursion relation for $\langle S(r)\rangle_Y$:

$$
\langle S(r)\rangle_{Y+dy} = (1 - \lambda\,dy)\langle S(r)\rangle_Y + \lambda\,dy \int d^2z\,\rho(z,r)\langle S(z)S(r-z)\rangle_Y .
\tag{3}
$$

The limit $dy \to 0$ yields

$$
\partial_Y \langle S(r)\rangle_Y = \frac{\bar{\alpha}}{2\pi} \int d^2z \frac{r^2}{z^2(r-z)^2} \left(\langle S(r)\rangle_Y - \langle S(z)S(r-z)\rangle_Y\right) .
\tag{4}
$$

We have used the known QCD result for the dipole splitting probability [3]

$$
dy\,\lambda \times \rho(z,r)\,d^2z = dy \frac{\bar{\alpha}}{2\pi} \frac{r^2}{z^2(r-z)^2} d^2z ,
\tag{5}
$$

where $\bar{\alpha} = \alpha_s N_c / \pi$. The latter equation is not closed: it involves the correlator $\langle S(z)S(r-z)\rangle_Y$. It turns out to be the first equation of an infinite hierarchy named after Balitskiĭ. A mean field approximation $\langle S(z)S(r-z)\rangle \simeq \langle S(z)\rangle\langle S(r-z)\rangle$ casts Eq.(4) into a closed form, known as the Balitskiĭ-Kovchegov (BK) equation [4].

Note that the fact that S is a scattering matrix element plays no special role in the above derivation. The only assumption is that the created dipoles are independent in such a way that their interaction factorizes. Thus we see that the quantity $\langle S^2(r)\rangle_Y$ obeys exactly the same evolution equation (4), with S replaced by S^2.

The BK equation falls in the universality class of the Fisher-Kolmogorov equation, and was shown [5] to admit traveling wave solutions at large Y, that decay

like $1 - \langle S(r) \rangle_Y \sim 1 - \langle S^2(r) \rangle_Y \sim (rQ_s(Y))^{2\gamma_0}$ for $r \ll 1/Q_s^2(Y)$ ($\gamma_0 \sim 0.63$ is fixed). $Q_s(Y)$ is the saturation scale. From Eq.(1), it results that the ratio

$$\frac{\sigma_{diff}}{\sigma_{tot}} = \frac{1 - 2\langle S(r) \rangle_{Y-Y_0} + \langle S^2(r) \rangle_{Y-Y_0}}{2(1 - \langle S(r) \rangle_{Y-Y_0})} \ , \tag{6}$$

which involves quantities that all obey the same equation, is independent of Y. A similar result was obtained by a direct calculation in Ref.[6].

The independence of that ratio with respect to the center-of-mass energy was seen experimentally in the HERA data. However, it seems to hold in rather narrow bins of the invariant diffracted mass. Here, we have shown that the total diffractive cross section (with a minimum rapidity gap Y_0) is independent of Y, which is a weaker result: it just means that at very high energy, deep inelastic scattering off a proton and off a Pomeron have the same energy dependence.

2 High mass diffraction

We compute the diffractive cross section to order α_s, allowing for at most one gluon in the diffractive system. Beyond that one-fluctuation, the S-matrix is approximated by its average $S(z) \sim \langle S(z) \rangle$. Replacing Eq.(4) for $\langle S \rangle$ and $\langle S^2 \rangle$ resp. into Eq.(1) and setting $Y_0 = Y_\mathbb{P}$ the size of the rapidity gap, one gets [7,1]

$$\frac{d\sigma_{diss}}{d^2b\, d\log(1/x_g)} = \frac{\bar{\alpha}}{2\pi} \int d^2z \frac{r^2}{z^2(r-z)^2}$$
$$\times \left(\mathcal{S}(Y_\mathbb{P}, z, b - (r-z)/2)\mathcal{S}(Y_\mathbb{P}, r-z, b-z/2) - \mathcal{S}(Y_\mathbb{P}, r, b) \right)^2 \tag{7}$$

where x_g is the longitudinal momentum fraction carried by the diffracted gluon. It is related to the rapidity variables through $\log(1/x_g) = Y - Y_\mathbb{P}$, and to the diffracted mass by $M_X = Q/\sqrt{x_g}$. $\mathcal{S}$ is the dipole elastic S-matrix element in the mean field approximation.

To arrive at HERA phenomenology, one has to convolute Eq.(7) with the distribution of dipoles of size r in the virtual photon, given by the squared wave function $|\psi(z_q, r)|^2$ for the splitting $\gamma^* \to q\bar{q}$ (x_g is the longitudinal momentum fraction carried by the quark). We also assume the Golec-Biernat-Wüsthoff (GBW) model [8] for $\mathcal{S}$, which in particular has no b-dependence. After some algebra, one gets

$$\frac{d\sigma_{diss}}{dM_X} = \frac{2\alpha_s N_c}{\pi^2} \frac{\sigma_0}{M_X} \int d^2r \int_0^1 dz_q |\psi(z_q, r)|^2$$
$$\times \int d^2z \frac{r^2}{z^2(r-z)^2} (\mathcal{S}(Y_\mathbb{P}, z)\mathcal{S}(Y_\mathbb{P}, r-z) - \mathcal{S}(Y_\mathbb{P}, r))^2 \ . \tag{8}$$

σ_0 is a parameter, and two more parameters enter $\mathcal{S}$. All of them are completely determined by the GBW fit to the F_2 structure function [8].

The agreement with the data so far published by the ZEUS collaboration [9] is good, see Fig.1. We expect that much could be learned on the saturation regime of QCD from observables such as $d\sigma_{diss}/d\log Q^2$ if that could be measured in the same kinematical regime [10]. We refer the reader to Ref.[1] for further details.
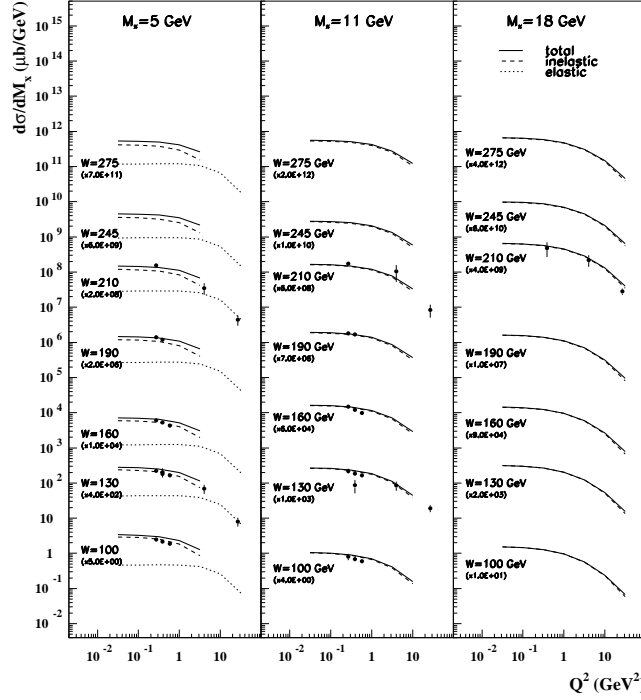
Figure 1. Comparison of the model with the data. Dashed line: Eq.(8). Dotted line: elastic component, taken from GBW. Full line: total. Data points are from the ZEUS coll.

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